Rough Outline of the Material in L1

1) I started from the Roman numerals and the Indo-Arabic numerals (numeral = name of number. A "numeral" is not a number, just the name of it).

E.g. IX versus 9, CCLVI versus 256

2) The Roman numeral system is difficult to use, especially when you try to do +,-,x and division with them.

One special feature of this system is that they don't have the notion of "zero" (which stands actually for "nothing", i.e. it is a "place holder").

So to express 10, you have to invent a symbol, which in this case is X.

Another feature is that the position to the left and to the right matters.

So IX means 10-1 which is 9, where XI means 10+1 which is 11.

3) Then I went on to talk about "long division". I asked students to compute 256/12 without using calculator.

Reason: To understand what this long division procedure means.

And also what "tools" is used when doing long division.

Answer: Two things at least. (i) 256 stands for 2x10^2 + 5x10^1 + 6x10^0

(if by 256 we mean the digital representation of a number)

(ii) Long division used the knowledge of "multiplication table"

4) To further understand the long division method, I asked the class to do base 7 long division, i.e.

256 understood as $2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

We want to divide it by 12, i.e. $1 \times 7^1 + 2 \times 7^0$

How does one proceed?

5) This brings us to to the concept of "set of integers", "set rational numbers" etc. and finally to the concept of "group".

(Here I was a little sloppy, so I'll repeat this point a little bit tomorrow).

A group is a mathematical system containing two things, i.e.

(a) a set (such as set of integers);

(b) an operation, e.g. + or x

One requires two things (at least) to happen:

(c) If x and y are two objects in the set, then x+y (or x multiplied by y) must be an object in the set too;

(d) For each x, there is a "reverse" object making things like this to happen: x+ reverse(x) = 0 or (for multiplication) x multiplied by "reverse(x)" = 1

Here the object 0 is a "neutral" object with respect to addition, because x+0 = x always.

Similarly, 1 is a "neutral" object with respect to multplication.

**** Important Remark ****

The property (c) above is called "closure".

Observation 1: The usual multiplication table we learned in school for numbers from 1 to 10 is not "closed"!

Observation 2: If we include more integer, it will be closed.

Observation 3: There are systems which are closed under some operation.

E.g.1 Consider the set of objects like E and O (for "even number" and "odd number") and the operation +

Then we have

+	E	O
E	E	O
O	O	E

E.g.2 Clock arithmetic (with only two hours i.e. 0 and 1)

That is whenever you have 2, 4, 6, they are all understood as 0.
Similarly 1,3,5,7 are all understood as 1

+	0	1
0	0	1
1	1	0

Such systems are useful in many areas. One of them is the "check digit" in the HK Id card, i.e. the last digit of your ID number enclosed by two brackets.

Final Remark: Group is useful in many many more areas and formed the basis of the proof by Abel and Galois that an equation like

$a \times x^5 + b \times x^4 + c \times x^3 + d \times x^2 + e \times x^1 + f \times x^0$ =0 where $a, b, c, \cdots$ are real numbers

may not have answer given in the form "using only +,-,x,division, taking roots (finitely many) times".

This phenomenon was surprising, because we learned in school that there is such a formula for the quadratic equation

$ax^2 + bx + c = 0$. The answer is as you've learned in schools, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$